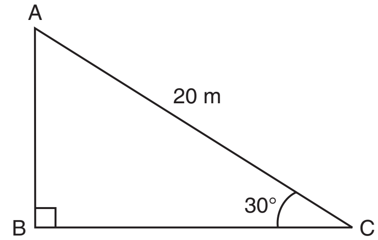
Some Applications of Trigonometry

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 9.1

Q. 1. *A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see figure).*

Sol. In the figure, let AC is the rope and AB is the pole. In right Δ ABC, we have:



$$\frac{AB}{AC} = \sin 30°$$

But $\sin 30° = \frac{1}{2}$

$$\Rightarrow \quad \frac{AB}{AC} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{AB}{20} = \frac{1}{2} \qquad [\because AC = 20\ m]$$

$$\Rightarrow \quad AB = 20 \times \frac{1}{2} = 10 \text{ m}$$

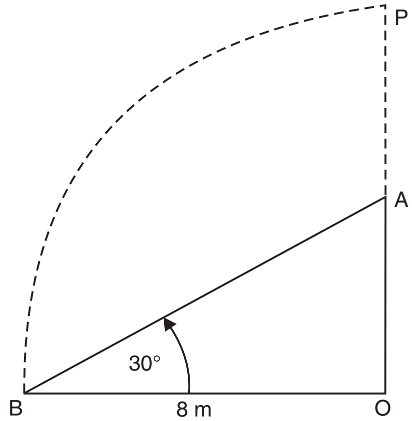
Thus, the required height of the pole is **10 m**.

Q. 2. *A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.* *[CBSE 2012]*

Sol. Let the original height of the tree = OP.

It is broken at A and its top is touching the ground at B.

Now, in right Δ AOB, we have



$$\frac{AO}{OB} = \tan 30°$$

But $\tan 30° = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad \frac{AO}{OB} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{AO}{8} = \frac{1}{\sqrt{3}} \Rightarrow AO = \frac{8}{\sqrt{3}}$$

Also, $\frac{AO}{OB} = \sec 30°$

$$\Rightarrow \quad \frac{AB}{8} = \frac{2}{\sqrt{3}} \quad \Rightarrow \quad AB = \frac{2 \times 8}{\sqrt{3}} = \frac{16}{\sqrt{3}}$$

Now, height of the tree

$$OP = OA + AP = OA + AB$$

$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} \qquad [\because AB = AP]$$

$$= \frac{24}{\sqrt{3}}\text{ m} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\text{ m} = \mathbf{8\sqrt{3}\ m}$$

Q. 3. *A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for older children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?*

Sol. In the figure, DE is the slide for younger children whereas AC is the slide for older children.

In right $\Delta\, ABC$,

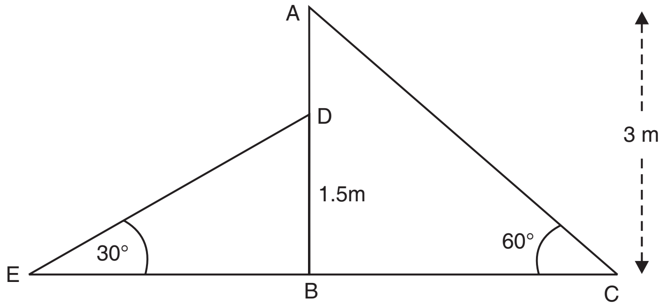
$$AB = 3 \text{ m}$$

$$AC = \text{length of the slide}$$

$$\therefore \quad \frac{AB}{AC} = \sin 60^\circ$$

$$\Rightarrow \quad \frac{3}{AC} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad AC = \frac{2 \times 3}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$



Again in right ΔBDE,

$$\frac{DE}{BD} = \text{cosec } 30^\circ = 2$$

$$\Rightarrow \quad \frac{DE}{1.5} = 2$$

$$\Rightarrow \quad DE = 2 \times 1.5 \text{ m}$$

$$\Rightarrow \quad DE = 3 \text{ m}$$

Thus, the lengths of slides are **3 m** and $\mathbf{2\sqrt{3}\ m}$.

Q. 4. *The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.*

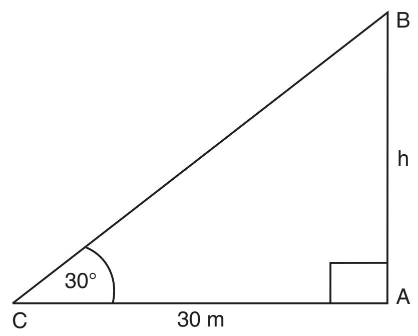
Sol. In right $\Delta\, ABC$, AB = the height of the tower. The point C is 30 m away from the foot of the tower,

$$\therefore \qquad AC = 30 \text{ m}$$

Now, $$\frac{AB}{AC} = \tan 30°$$

$$\Rightarrow \qquad \frac{h}{30} = \frac{1}{\sqrt{3}} \qquad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \qquad h = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$$



Thus, the required height of the tower is $\mathbf{10\sqrt{3}}$ **m**.

Q. 5. *A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.*

Sol. Let in the right Δ *AOB,*

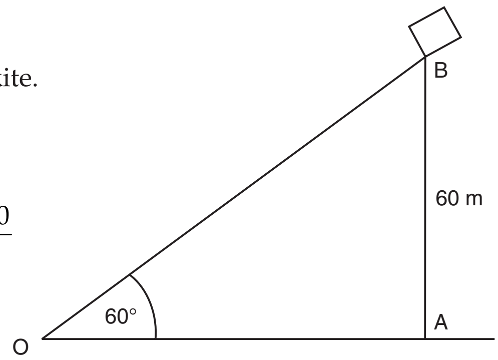
$$OB = \text{Length of the string}$$

$$AB = 60 \text{ m} = \text{Height of the kite.}$$

$$\therefore \qquad \frac{OB}{AB} = \operatorname{cosec} 60° = \frac{2}{\sqrt{3}}$$

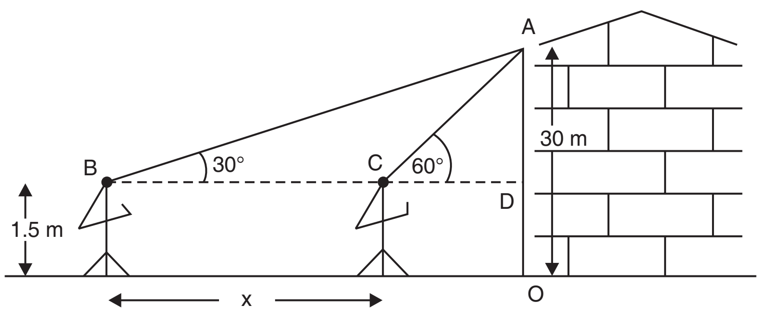
$$\Rightarrow \qquad \frac{OB}{60} = \frac{2}{\sqrt{3}} \quad \Rightarrow \quad OB = \frac{2 \times 60}{\sqrt{3}}$$

$$\Rightarrow \qquad OB = \frac{120 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 40\sqrt{3}$$



Thus, length of the string is $\mathbf{40\sqrt{3}}$ **m**.

Q. 6. *A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.*



Sol. Here, *OA* is the building.

In right Δ *ABD,*

$$\frac{AD}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BD = AD\sqrt{3} = 28.5\sqrt{3} \qquad [\because AD = 30\ m - 1.5\ m = 28.5\ m]$$

Also, in right $\Delta\, ACD$,

$$\frac{AD}{CD} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad CD = \frac{AD}{\sqrt{3}} = \frac{28.5}{\sqrt{3}}$$

Now, $$BC = BD - CD = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$$

$$\Rightarrow \quad BC = 28.5\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right]$$

$$= 28.5\left[\frac{3-1}{\sqrt{3}}\right]$$

$$= 28.5 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{28.5 \times 2 \times \sqrt{3}}{3}$$

$$= 9.5 \times 2 \times \sqrt{3}$$

$$= \mathbf{19\sqrt{3}\ m}$$

Thus the distance walked by the man towards the building = $19\sqrt{3}$ m.

Q. 7. *From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.* (CBSE 2010)

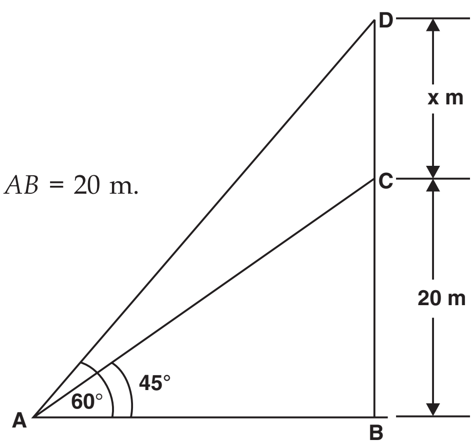
Sol. Let the height of the building be BC

$$\therefore \quad BC = 20 \text{ m}$$

And height of the tower be CD.

Let the point A be at a distance y metres from the top B of the building.

Now, in right $\Delta\, ABC$,



$$\frac{BC}{AB} = \tan 45° = 1$$

$$\Rightarrow \quad \frac{20}{y} = 1 \Rightarrow y = 20 \text{ m} \quad i.e., \quad AB = 20 \text{ m}.$$

Now, in right $\Delta\, ABD$,

$$\frac{BD}{AB} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad \frac{BD}{20} = \sqrt{3}$$

$$\Rightarrow \quad \frac{20+x}{20} = \sqrt{3} \quad \Rightarrow \quad 20 + x = 20\sqrt{3}$$

$$\Rightarrow \quad x = 20\sqrt{3} - 20 = 20\,[\sqrt{3} - 1]$$

$$\Rightarrow \quad x = 20\,[1.732 - 1]$$

$$\Rightarrow \quad x = 20 \times 0.732 = 14.64$$

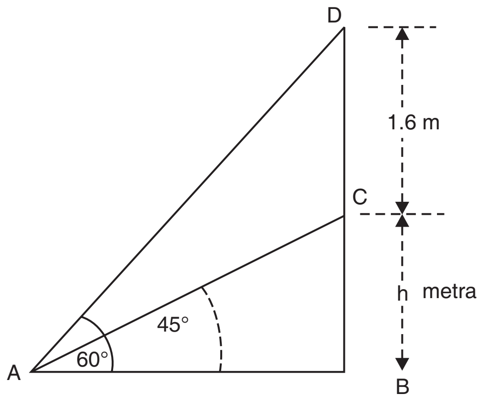
Thus, the height of the tower is **14.64 m.**

Q. 8. *A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.* (CBSE 2012)

Sol. In the figure,

DC represents the statue.

BC represents the pedestal.



Now in right $\Delta\, ABC$, we have

$$\frac{AB}{BC} = \cot 45° = 1$$

$$\Rightarrow \quad \frac{AB}{h} = 1 \quad \Rightarrow \quad AB = h \text{ metres.}$$

Now in right $\Delta\, ABD$, we get

$$\frac{BD}{AB} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad BD = \sqrt{3} \times AB = \sqrt{3} \times h$$

$$\Rightarrow \quad h + 1.6 = \sqrt{3}\, h$$

$$\Rightarrow \quad \frac{h+1.6}{h} = \sqrt{3} \quad \Rightarrow \quad h(\sqrt{3} - 1) = 1.6$$

$$\Rightarrow \quad h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$h = \frac{1.6}{3-1} \times (\sqrt{3}+1)$$

$$= \frac{1.6}{2} \times \sqrt{3}+1$$

$$= 0.8\,(\sqrt{3} + 1) \text{ m}$$

Thus, the height of the pedestal = **0.8 ($\sqrt{3}$ + 1) m.**

Q. 9. *The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.* (CBSE Delhi 2014)

Sol. In the figure, let height of the building = $AB = h$ m

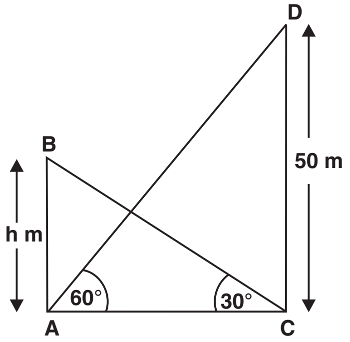
Let CD be the tower.

$\therefore \qquad CD = 50$ m

Now, in right Δ *BAC*,

$$\frac{AC}{AB} = \cot 30° = \sqrt{3}$$

$$\Rightarrow \qquad \frac{AC}{h} = \sqrt{3} \quad \Rightarrow \quad AC = h\sqrt{3} \qquad ...(1)$$



Again, in right Δ *DCA*,

$$\frac{DC}{AC} = \tan 60°$$

$$\Rightarrow \qquad \frac{50}{AC} = \sqrt{3} \quad \Rightarrow \quad AC = \frac{50}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2),

$$\sqrt{3}\,h = \frac{50}{\sqrt{3}}$$

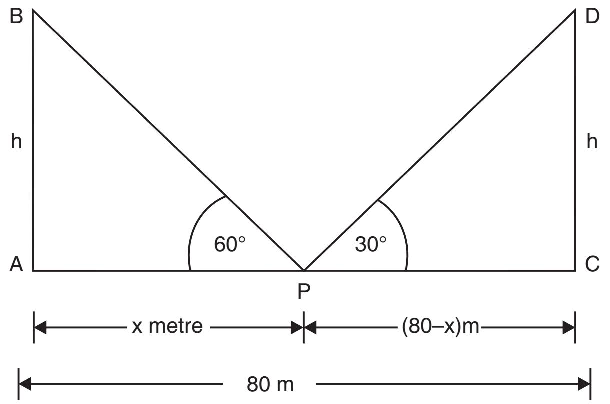
$$\Rightarrow \qquad h = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3}$$

Thus, the height of the building = $\mathbf{16\frac{2}{3}}$ **m**

Q. 10. *Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.* (CBSE 2012)

Sol. Let *AB* and *CD* are the two poles such that:

$$AB = h \text{ metres}$$
$$CD = h \text{ metres}$$



Let '*P*' be the point on the road such that

$$AP = x \text{ m}$$
$$CP = (80 - x) \text{ m}$$

Now, in right $\Delta\, APB$, we have

$$\frac{AB}{AP} = \tan 60°$$

$$\Rightarrow \quad \frac{h}{x} = \sqrt{3} \quad \Rightarrow \quad h = x\sqrt{3} \qquad \text{...(1)}$$

Again in right $\Delta\, CPD$,

$$\frac{CD}{CP} = \tan 30°$$

$$\Rightarrow \quad \frac{h}{(80-x)} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{80-x}{\sqrt{3}} \qquad \text{...(2)}$$

From (1) and (2), we get

$$\sqrt{3}\, x = \frac{80-x}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3} \times \sqrt{3} \times x = 80 - x$$

$$\Rightarrow \quad 3x = 80 - x$$

$$\Rightarrow \quad 3x + x = 80$$

$$\Rightarrow \quad 4x = 80$$

$$\Rightarrow \quad x = \frac{80}{4} = 20$$

$$\Rightarrow \quad 80 - x = 80 - 20 = 60$$

Now, from (1), we have:

$$h = \sqrt{3} \times 20 = 1.732 \times 20$$

$$= 34.64$$

Thus, (*i*) The required point is **20 m** away from the first pole and **60 m** away from the second pole.

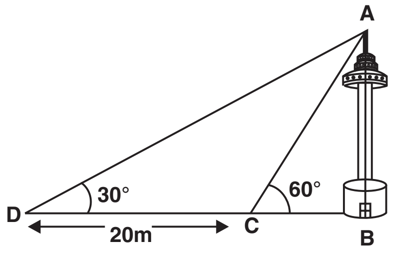
(*ii*) Height of each pole = **34.64 m.**

Q. 11. *A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see figure). Find the height of the tower and the width of the canal.*

Sol. Let the TV Tower be $AB = h$ m.

Let the point 'C' be such that

$$BC = x \text{ and } CD = 20 \text{ m}.$$



Now, in right $\Delta\, ABC$, we have:

$$\frac{AB}{BC} = \tan 60°$$

$$\Rightarrow \quad \frac{h}{x} = \sqrt{3} \quad \Rightarrow \quad h = \sqrt{3}\, x \qquad \text{...(1)}$$

In right $\Delta\, ABD$, we have:

$$\frac{AB}{BD} = \tan 30°$$

$$\Rightarrow \quad \frac{h}{x+20} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{x+20}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2), we get

$$\sqrt{3}\,x = \frac{x+20}{\sqrt{3}} \quad \Rightarrow \quad 3x = x + 20$$

$$\Rightarrow \quad 3x - x = 20$$

$$\Rightarrow \quad 2x = 20 \quad \Rightarrow \quad x = \frac{20}{2} = 10 \text{ m}$$

Now, from (1), we get

$$h = \sqrt{3} \times 10 = 1.732 \times 10 = 17.32$$

Thus, the height of the tower = **17.32 m.**

Also width of the river = **10 m.**

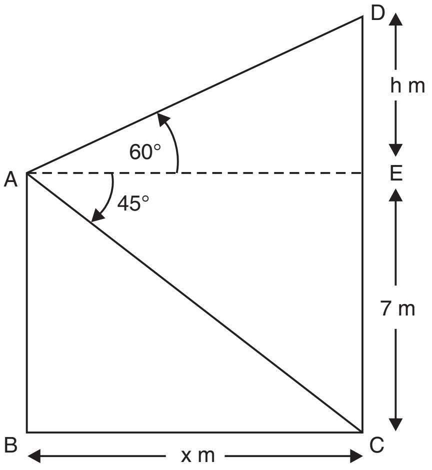
Q. 12. *From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.*

Sol. In the figure, let AB be the height of the tower.

$\therefore$ AB = 7 metres.

Let CD be the cable tower.

$\therefore$ In right Δ DAE, we have



$$\frac{DE}{EA} = \tan 60°$$

$$\Rightarrow \quad \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow \quad h = \sqrt{3}\,.\,x \qquad ...(1)$$

Again, in right Δ ABC,

$$\frac{AB}{BC} = \tan 45°$$

$$\Rightarrow \quad \frac{7}{x} = 1$$

$$\Rightarrow \quad x = 7 \qquad ...(2)$$

From (1) and (2),

$$h = 7\sqrt{3} = DE$$

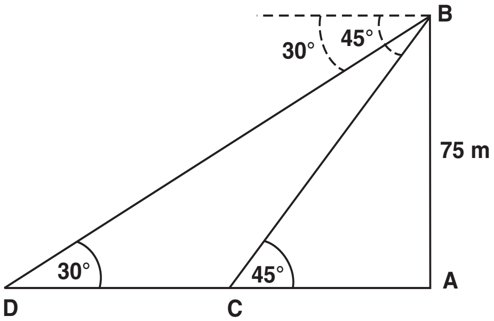
$$\therefore \quad CD = CE + ED$$

$$= 7 + 7\sqrt{3} = 7\,(1 + \sqrt{3}) \text{ m}$$

$$= 7\,(1 + 1.732) \text{ m} = 7 \times 2.732 \text{ m} = 19.124 \text{ m}$$

Thus, the height of the cable tower is **19.124 m.**

Q. 13. *As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.*



Sol. In the figure, let AB represent the light house.

$\therefore \quad AB = 75$ m.

Let the two ships be C and D such that angles of depression from B are 45° and 30° respectively.

Now in right $\Delta\, ABC$, we have:

$$\frac{AB}{AC} = \tan 45°$$

$$\Rightarrow \quad \frac{75}{AC} = 1 \quad \Rightarrow \quad AC = 75 \qquad ...(1)$$

Again, in right $\Delta\, ABD$, we have:

$$\frac{AB}{AD} = \tan 30°$$

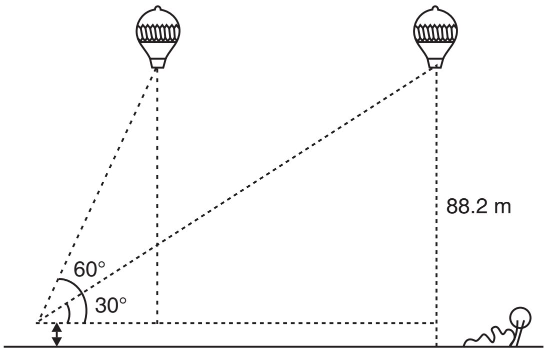
$$\Rightarrow \quad \frac{75}{AD} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad AD = 75\sqrt{3} \qquad ...(2)$$

Since the distance between the two ships $= CD$

$$= AD - AC$$
$$= 75\sqrt{3} - 75 = 75[\sqrt{3} - 1]$$
$$= 75[1.732 - 1] = 75 \times 0.732 = 54.9$$

Thus, the required distance between the ships = **54.9 m.**

Q. 14. *A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (see figure). Find the distance travelled by the balloon during the interval.* (AI CBSE 2009)



Sol. In the figure, let C be the position of the observer (the girl).

A and P are two positions of the balloon.

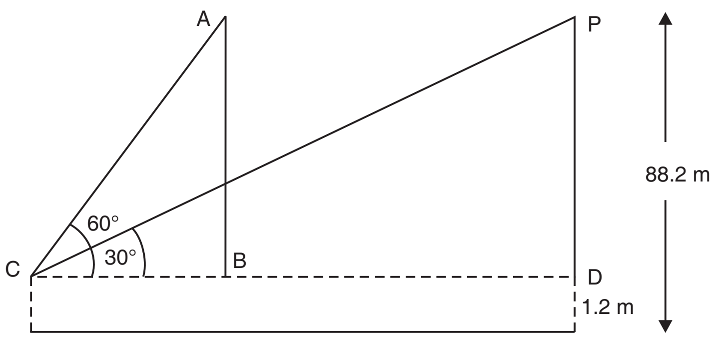
CD is the horizontal line from the eyes of the (observer) girl.

Here $PD = AB = 88.2 \text{ m} - 1.2 \text{ m} = 87 \text{ m}$

In right $\Delta\, ABC$, we have

$$\frac{AB}{BC} = \tan 60°$$

$$\Rightarrow \quad \frac{87}{BC} = \sqrt{3} \quad \Rightarrow \quad BC = \frac{87}{\sqrt{3}} \text{ m}$$



In right $\Delta\, PDC$, we have

$$\frac{PD}{CD} = \tan 30°$$

$$\Rightarrow \quad \frac{87}{CD} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad CD = 87\sqrt{3}$$

Now, $$BD = CD - BC$$

$$= 87\sqrt{3} - \frac{87}{\sqrt{3}}$$

$$= 87\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = 87 \times \left(\frac{3-1}{\sqrt{3}}\right) = \frac{2 \times 87}{\sqrt{3}} \text{ m}$$

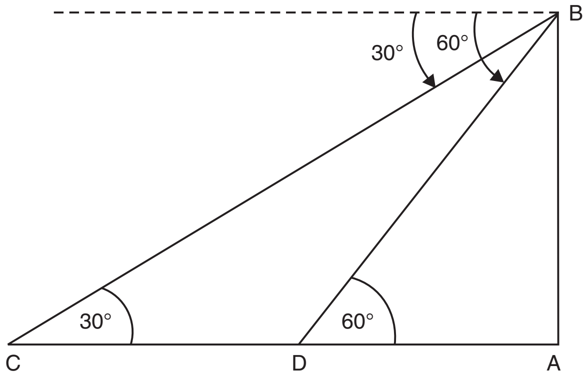
$$= \frac{2 \times 87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2 \times 87 \times \sqrt{3}}{3} = 2 \times 29 \times \sqrt{3} \text{ m}$$

$$= 58\sqrt{3} \text{ m}$$

Thus, the required distance between the two positions of the balloon = **$58\sqrt{3}$ m**

Q. 15. *A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.* (CBSE 2009)

Sol. In the figure, let AB is the height of the tower and C and D be the two positions of the car.



In right $\Delta\ ABD$, we have:

$$\frac{AB}{AD} = \tan 60°$$

$$\Rightarrow \qquad \frac{AB}{AD} = \sqrt{3} \quad \Rightarrow \quad AB = \sqrt{3} \cdot AD \qquad ...(1)$$

In right $\Delta\ ABC$, we have:

$$\frac{AB}{AC} = \tan 30°$$

$$\Rightarrow \qquad \frac{AB}{AC} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad AB = \frac{AC}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2)

$$\sqrt{3}\ AD = \frac{AC}{\sqrt{3}}$$

$$\Rightarrow \qquad AC = \sqrt{3} \times \sqrt{3} \times AD = 3\ AD$$

Now $\qquad CD = AC - AD$

$$= 3\ AD - AD = 2\ AD$$

Since the distance $2\ AD$ is covered in 6 seconds,

$\therefore$ The distance AD will be covered in $\frac{6}{2}$ *i.e.,* 3 seconds.

Thus, the time taken by the car to reach the tower from D is **3 seconds**.

Q. 16. *The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.*

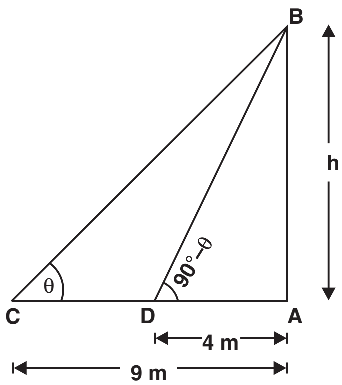
Sol. Let the tower be represented by AB in the figure.

Let $\qquad AB = h$ metres.

$\therefore$ In right $\Delta\ ABC$, we have:

$$\frac{AB}{AC} = \tan\theta$$

$$\Rightarrow \quad \frac{h}{9} = \tan\theta \qquad \text{...(1)}$$



In right $\Delta\, ABD$, we have:

$$\frac{AB}{AD} = \tan(90° - \theta) = \cot\theta$$

$$\Rightarrow \quad \frac{h}{4} = \cot\theta \qquad \text{...(2)}$$

Multiplying (1) and (2), we get

$$\frac{h}{9} \times \frac{h}{4} = \tan\theta \times \cot\theta = 1 \qquad [\because \tan\theta \times \cot\theta = 1]$$

$$\Rightarrow \quad \frac{h^2}{36} = 1 \quad \Rightarrow \quad h^2 = 36$$

$$\Rightarrow \quad h = \pm\, 6 \text{ m}$$

$$\Rightarrow \quad h = 6 \text{ m} \qquad [\because \text{Height is positive only}]$$

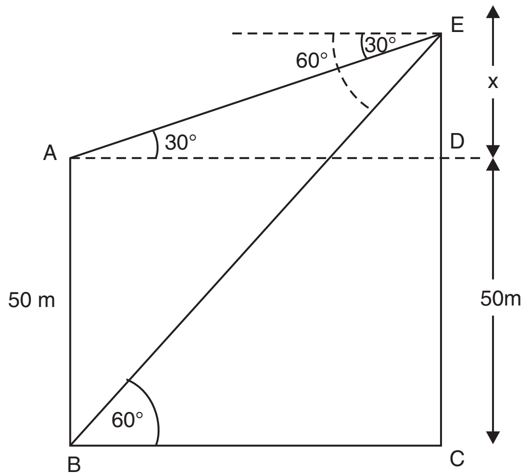
Thus, the height of the tower is **6 m.**

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 9.2

Q. 1. *The angles of depression of the top and the bottom of a building 50 m high as observed from the top of a tower are 30° and 60° respectively. Find the height of the tower and also the horizontal distance between the building and the tower.*

Sol. In the figure,



Let AB = 50 m be the building.

Let CE be the tower such that $CE = (50 + x)$ m

In right Δ ADE, we have:

$$\frac{DE}{AD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{x}{AD} = \frac{1}{\sqrt{3}} \Rightarrow AD \Rightarrow x\sqrt{3} \quad \text{or} \quad BC = x\sqrt{3} \qquad ...(1)$$

In right Δ ACE, we have:

$$\frac{CE}{BC} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad \frac{50+x}{BC} = \sqrt{3} \Rightarrow BC = \frac{50+x}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2), we get

$$\sqrt{3}\,x = \frac{50+x}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}\,x \times \sqrt{3} = 50 + x$$

$$\Rightarrow \quad 3x - x = 50 \Rightarrow x = 25$$

$\therefore$ Height of the tower $= 50 + x$

$= 50 + 25$

$=$ **75 m**

Now from (1), $BC = \sqrt{3} \times x$

$= \sqrt{3} \times 25$ m

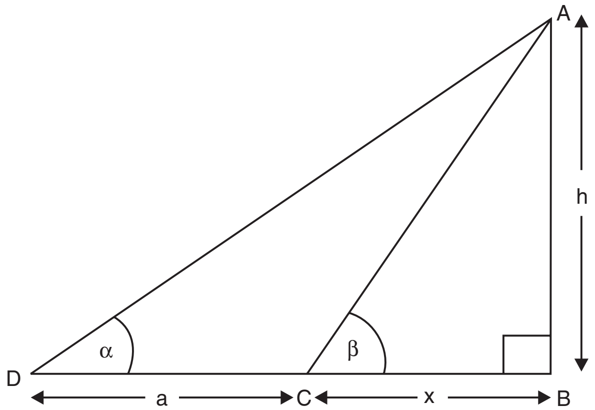
$= 1.732 \times 25$ m

$=$ **43.25 m**

i.e., The horizontal distance between the building and the tower = **43.25 m.**

Q. 2. *The angle of elevation of the top of a tower as observed from a point on the ground is 'α' and on moving 'a' metres towards the tower, the angle of elevation is 'β'. Prove that the height of the tower is* $\dfrac{a\tan\alpha\cdot\tan\beta}{\tan\beta-\tan\alpha}$.

Sol. In the figure, let the tower be represented by AB.



$\therefore$ In right Δ ABC, we have:

$$\tan\beta = \frac{AB}{BC} = \frac{h}{x}$$

$$\Rightarrow \quad x\tan\beta = h$$

$$\Rightarrow \quad x = \frac{h}{\tan\beta} \qquad ...(1)$$

Now, in right Δ ABD, we have:

$$\frac{AB}{BD} = \tan\alpha$$

$$\Rightarrow \quad \frac{h}{x+a} = \tan\alpha$$

$$\Rightarrow \quad h = (x + a)\tan\alpha$$

$$\Rightarrow \quad h = x\tan\alpha + a\tan\alpha$$

$$\Rightarrow \quad h = \frac{h}{\tan\beta}\cdot\tan\alpha + a\tan\alpha \qquad \left[\because\ x = \frac{h}{\tan\beta}\ \textit{from (1)}\right]$$

$$\Rightarrow \quad h = \frac{h\tan\alpha + a\tan\alpha\,.\tan\beta}{\tan\beta}$$

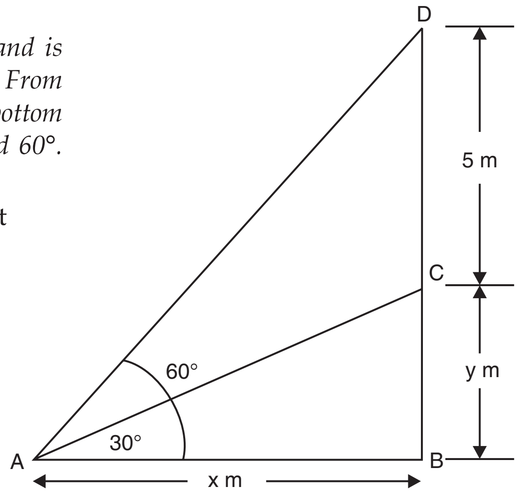
$$\Rightarrow \quad h\tan\beta = h\tan\alpha + a\tan\alpha\cdot\tan\beta$$

$$\Rightarrow \quad h\tan\beta - h\tan\alpha = a\tan\alpha\cdot\tan\beta$$

$$\Rightarrow \quad h(\tan\beta - \tan\alpha) = a\tan\alpha\cdot\tan\beta$$

$$\Rightarrow \quad h = \frac{a\tan\alpha\cdot\tan\beta}{\tan\beta - \tan\alpha}$$

Q. 3. *A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height 5 m. From a point on the plane the angles of elevation of the bottom and top of the flag staff are respectively 30° and 60°. Find the height of the tower.*



Sol. Let in the figure, BC be the tower such that

$$BC = y \text{ metres.}$$

CD be the flag staff such that

$$CD = 5 \text{ m}$$

$$\Rightarrow \quad BD = (y + 5) \text{ m.}$$

In right $\Delta\,ABC$, we have:

$$\frac{BC}{AB} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{y}{x} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad x = \sqrt{3}\,.\,y \qquad \text{...(1)}$$

In right $\Delta\,ABD$, we have:

$$\frac{BD}{AB} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad \frac{(y+5)}{x} = \sqrt{3} \quad \Rightarrow \quad y + 5 = \sqrt{3}\,x$$

$$\therefore \quad y + 5 = \sqrt{3}\,(\sqrt{3}\ y) \qquad [x = \sqrt{3}\cdot y \text{ from (1)}]$$

$$\Rightarrow \quad y + 5 = 3y$$

$$\Rightarrow \quad 3y - y = 5 \quad \Rightarrow \quad y = \frac{5}{2} = 2.5 \text{ m}$$

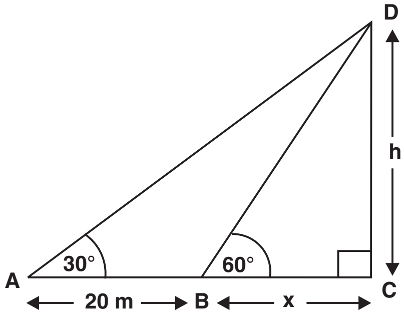
$\therefore$ The height of the tower = **2.5 m.**

Q. 4. *The length of the shadow of a tower standing on level plane is found to be 20 m longer when the sun's altitude is 30° than when it was 60°. Find the height of the tower.*

Sol. In the figure, let CD be the tower such that

$$CD = h \text{ metres}$$

Also $\quad BC = x$ metres



In right $\Delta\, BCD$, we have:

$$\frac{CD}{BC} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad \frac{h}{x} = \sqrt{3} \quad \Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad ...(1)$$

In right $\Delta\, ACD$, we have:

$$\frac{CD}{AC} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{h}{20+x} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \sqrt{3}\,h = 20 + x$$

$$\Rightarrow \quad \sqrt{3}\,h = 20 + \frac{h}{\sqrt{3}} \qquad \left[\textit{From (1)}, x = \frac{h}{\sqrt{3}}\right]$$

$$\Rightarrow \quad \sqrt{3} \times \sqrt{3}\,h = 20\sqrt{3} + h$$

$$\Rightarrow \quad 3h - h = 20\sqrt{3}$$

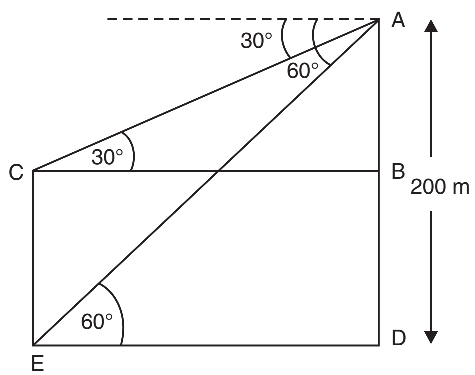
$$\Rightarrow \quad 2h = 20\sqrt{3} \quad \Rightarrow \quad h = \frac{20}{2}\sqrt{3} = 10\sqrt{3}$$

$$\Rightarrow \quad h = 10 \times 1.732 = 17.32 \text{ m}$$

Thus, the height of the tower = **17.32 m.**

Q. 5. *From the top of a hill 200 m high, the angles of depression of the top and bottom of a pillar are 30° and 60° respectively. Find the height of the pillar and its distance from the hill.* [CBSE 2014]

Sol. In the figure, let AD is the hill such that AD = 200 m and CE is the pillar.



In right $\Delta\, ADE$, we have:

$$\frac{AD}{DE} = \tan 60 = \sqrt{3}$$

$$\therefore \quad \frac{200}{DE} = \sqrt{3}$$

$$\Rightarrow \quad DE = \frac{200}{\sqrt{3}} = \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \qquad DE = \frac{\sqrt{3} \times 200}{3} = \frac{1.73 \times 200}{3}$$

$$= \frac{346}{3} = 115.33 \text{ m}$$

$\Rightarrow$ Distance between pillar and hill = **115.33 m**

Now, $\qquad BC = DE = \frac{200}{\sqrt{3}}$ m $\qquad [\because DE = BC]$

In right Δ ABC, we have:

$$\frac{AB}{BC} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \qquad AB = \frac{BC}{\sqrt{3}} = \frac{200}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{200}{3} \qquad [\because BC = \frac{200}{\sqrt{3}}]$$

$$= 66.67 \text{ m}$$

$\therefore$ Height of the pillar

$$CE = AD - AB \qquad [\because CE = BD]$$

$$= 200 - 66.67 \text{ m}$$

$$= \mathbf{133.33 \text{ m}}$$

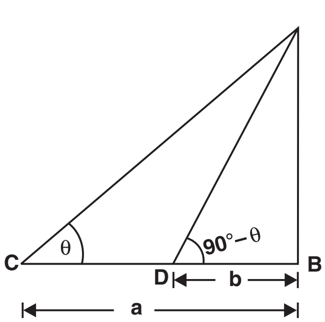
Q. 6. *The angles of elevation of the top of a tower from two points on the ground at distances a and b units from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is* $\sqrt{ab}$ *units.*

Sol. In the figure, AB is the tower, such that:

$$AB = h$$

$$BD = b$$

$$BC = a$$



In right Δ ABD, we have

$$\frac{AB}{BD} = \tan(90° - \theta)$$

$$\Rightarrow \qquad \frac{h}{b} = \tan(90° - \theta)$$

$$\Rightarrow \qquad h = b \cot \theta \qquad \text{...(1)}$$

In right Δ ABC, we have

$$\frac{AB}{BC} = \tan \theta$$

$$\Rightarrow \qquad \frac{h}{a} = \tan \theta \quad \Rightarrow \quad h = a \tan \theta \qquad \text{...(2)}$$

Multiplying (1) and (2), we get

$$h \times h = b \cot \theta \times a \tan \theta$$

$$\Rightarrow \qquad h^2 = a \times b \times (\cot \theta \times \tan \theta)$$

$$\Rightarrow \qquad h^2 = a\, b \qquad [\because \cot \theta \times \tan \theta = 1]$$

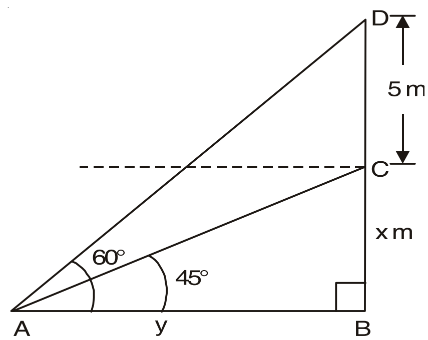
$$\Rightarrow \qquad \boldsymbol{h = \sqrt{a\, b}}$$

Q. 7. *A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point 'A' on the ground is 60° and the angle of depression of the point 'A' from the top of the tower is 45°. Find the height of the tower.* *[A.I. CBSE 2004]*

Sol. In the figure, let BC be the tower and CD be the pole.

Let $BC = x$ metres and $AB = y$ metres

In right $\Delta\, ABC$, we get



$$\frac{BC}{AB} = \tan 45° = 1$$

$$\Rightarrow \quad BC = AB \Rightarrow y = x \quad \text{... (1)}$$

In right $\Delta\, ABD$, we have:

$$\frac{BD}{AB} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad \frac{x+5}{y} = \sqrt{3}$$

$$\Rightarrow \quad y\sqrt{3} = x + 5$$

$$\Rightarrow \quad x\sqrt{3} = x + 5 \quad [\because x = y \text{ from (1)}]$$

$$\therefore \quad \sqrt{3}\,x - x = 5$$

$$\Rightarrow \quad (\sqrt{3} - 1)\,x = 5$$

$$\Rightarrow \quad x = \frac{5}{\sqrt{3}-1} = \frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{5(\sqrt{3}+1)}{3-1} = \frac{5(1.732+1)}{2}$$

$$= \frac{5}{2} \times 2.732 \text{ m}$$

$$= 5 \times 1.366 \text{ m}$$

$$= 6.83 \text{ m}$$

Thus, the height of the tower = **6.83 m**

MORE QUESTIONS SOLVED

I. SHORT ANSWER TYPE QUESTIONS

Q. 1. *A tower stands vertically on the ground. From a point on the ground which is 20 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower.* (CBSE 2010)

Sol. In the figure, AB is the tower,

$$\therefore \quad AB = h \text{ metres}$$

In rt $\Delta\, ABC$, we have:

$$\frac{BC}{AC} = \tan 60°$$

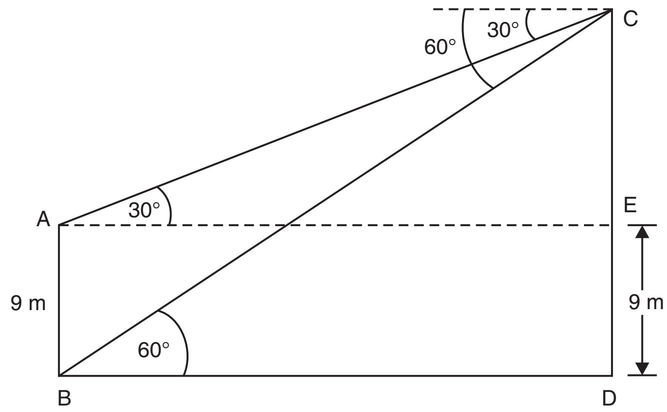
$$\Rightarrow \quad \frac{h}{20} = \sqrt{3}$$

$$| \because \tan 60° = \sqrt{3} \text{ and } AB = 20 \text{ m}$$

$$\Rightarrow \quad h = 20\sqrt{3} \text{ metre}$$

Thus, the height of the tower $= 20\sqrt{3}$ m.

Q. 2. *The angle of depression of the top and the bottom of a 9 m high building from the top of a tower are 30° and 60° respectively. Find the height of the tower and the distance between the building and the tower.*



Sol. Let AB represents the building and CD be the tower.

$\therefore \quad AB = 9$ m

In right Δ BDC, we have:

$$\frac{CD}{DB} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad CD = DB \cdot \sqrt{3} \quad \text{...(1)}$$

In right Δ AEC, we have:

$$\frac{CE}{AE} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{CD-9}{AE} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad AE = \sqrt{3}\ CD - 9\sqrt{3}$$

$$\Rightarrow \quad BD = \sqrt{3}\ (DB \cdot \sqrt{3}) - 9\sqrt{3}$$

$$\Rightarrow \quad BD = 3\ BD - 9\sqrt{3}$$

$$\Rightarrow \quad 2BD = 9\sqrt{3}$$

$$\Rightarrow \quad BD = \frac{9}{2}\sqrt{3} = \frac{9 \times 1.732}{2}$$

$$\Rightarrow \quad BD = 7.8 \text{ m}$$

From (1), we have,

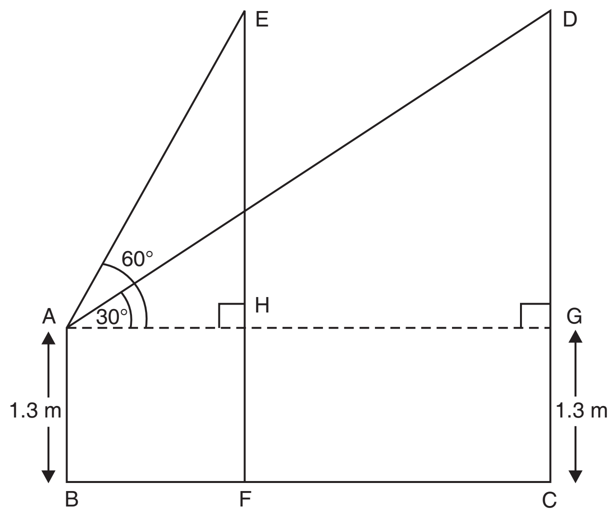
$$CD = \sqrt{3} \times \frac{9}{2} \times \sqrt{3} = \frac{27}{2} = 13.5$$

Thus, height of the tower = **13.5 m**

Distance between the building and the tower = **7.8 m**

II. LONG ANSWER TYPE QUESTIONS

Q. 1. *A boy whose eye level is 1.3 m from the ground, spots a balloon moving with the wind in a horizontal level at some height from the ground. The angle of elevation of the balloon from the eyes of the boy at any instant is 60°. After 2 seconds, the angle of elevation reduces to 30°. If the speed of the wind at that moment is $29\sqrt{3}$ m/s, then find the height of the balloon from ground.* (CBSE 2009 C)



Sol. Let E and D be the two positions of the balloon.

Let AB be the position of the boy.

$\therefore \quad AB = 1.3$ m

$\Rightarrow \quad HF = CG = 1.3$ m

Also speed of the wind = $29\sqrt{3}$ m/s

Distance covered by the balloon in 2 seconds

$$= ED = HG = 2 \times 29\sqrt{3} \text{ m}$$

$$= 58\sqrt{3} \text{ m}$$

$\therefore \quad AG = AH + HG$

$$= AH + 58\sqrt{3} \text{ m} \qquad \text{...(1)}$$

Now, in right $\Delta\, AEH$, we have

$$\frac{EH}{AH} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad EH = AH \cdot \sqrt{3} \quad \Rightarrow \quad AH = \frac{EH}{\sqrt{3}} \qquad \text{...(2)}$$

In right Δ AGD, we have

$$\frac{DG}{AG} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{DG}{(AH + 58\sqrt{3})} = \frac{1}{\sqrt{3}} \qquad \text{[From (1)]}$$

$$\Rightarrow \quad \sqrt{3}\,DG = AH + 58\sqrt{3}$$

$$\Rightarrow \quad \sqrt{3}\,DG = \frac{EH}{\sqrt{3}} + 58\sqrt{3} \qquad \text{[From (2)]}$$

$$\Rightarrow \quad \sqrt{3} \times \sqrt{3} \times DG = EH + 58 \times \sqrt{3} \times \sqrt{3}$$

$$\Rightarrow \quad 3\,DG = EH + 3 \times 58$$

$$\Rightarrow \quad 3\,DG = EH + 174$$

$$\Rightarrow \quad 3\,DG - EH = 174$$

$$\Rightarrow \quad 2\,DG = 174 \qquad [\because DG = EH]$$

$$\Rightarrow \quad DG = \frac{174}{2} = 87 \text{ m}$$

$$\therefore \quad CD = DG + GC = (87 + 1.3) \text{ m}$$

$$= 88.3 \text{ m}$$

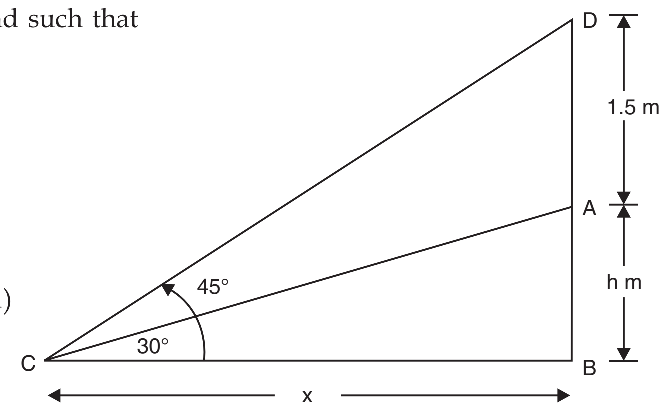
Thus, the height of the balloon = **88.3 m.**

Q. 2. *A statue, 1.5 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 45° and from the same point the angle of elevation of the top of the pedestal is 30°. Find the height of the pedestal from the ground.* (CBSE 2012, 2009-C)

Sol. Let AB be the pedestal and $AB = h$

Let C be the point on the ground such that

$$BC = x \text{ metres.}$$



In right Δ ACB, we have:

$$\frac{AB}{BC} = \tan 30°$$

$$\Rightarrow \quad \frac{h}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \sqrt{3}\,h \quad \text{...(1)}$$

In right Δ DCB, we have:

$$\frac{BD}{BC} = \tan 45°$$

$$\Rightarrow \quad \frac{BD}{x} = 1$$

$$\Rightarrow \quad \frac{AB + AD}{x} = 1$$

$$\Rightarrow \quad \frac{h+1.5}{x} = 1$$

$$\Rightarrow \quad h + 1.5 = x$$

$$\Rightarrow \quad h + 1.5 = \sqrt{3}\,h \qquad \text{[From (1)]}$$

$$\Rightarrow \quad \sqrt{3}\,h - h = 1.5$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 1.5$$

$$\Rightarrow \quad h = \frac{1.5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow \quad h = \frac{1.5(\sqrt{3}+1)}{3-1} = \frac{1.5(\sqrt{3}+1)}{2}\text{ m}$$

$$\Rightarrow \quad h = 0.75(\sqrt{3} + 1)\text{ m}$$

Thus, the height of the pedestal = **0.75 ($\sqrt{3}$ + 1) m.**

Q. 3. *The angles of depression of the top and battom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.* (CBSE 2009)

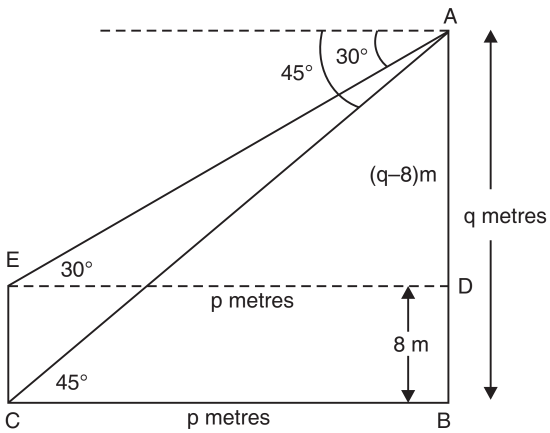
Sol. Let the multistoreyed building be AB.

$$\therefore \quad AB = q \text{ metres}$$

$$\Rightarrow \quad AD = (q - 8)\text{ m } [\because BD = 8\text{ m}]$$

Let EC be the small building.

Now, in right $\Delta\, ABC$, we have:



$$\frac{AB}{BC} = \tan 45° = 1$$

$$\Rightarrow \quad AB = BC$$

$$\Rightarrow \quad q = p \qquad ...(1)$$

In right $\Delta\, ADE$, we have:

$$\frac{AD}{DE} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}\,AD = DE$$

$$\Rightarrow \quad \sqrt{3}(q - 8) = p$$

$$\Rightarrow \quad \sqrt{3}\,q - 8\sqrt{3} = q \qquad \text{[From (1)]}$$

$$\Rightarrow \quad \sqrt{3}\,q - q = 8\sqrt{3}$$

$$\Rightarrow \quad q(\sqrt{3} - 1) = 8\sqrt{3}$$

$$\therefore \quad q = \frac{8\sqrt{3}}{\sqrt{3}-1}$$

$$\Rightarrow \quad q = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}\text{ m}$$

$$= \frac{8\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3})^2 - 1^2} \text{ m} = \frac{8\sqrt{3}(\sqrt{3}+1)}{2} \text{ m}$$

$$= 4(3+\sqrt{3}) \text{ m} = 4(3 + 1.732) \text{ m}$$

$$= 18.928 \text{ m}$$

Since $p = q$

$\Rightarrow$ $p = 18.928$ m

$\therefore$ Distance between the two buildings = **18.928 m**

Height of the multi-storeyed building = **18.928 m.**

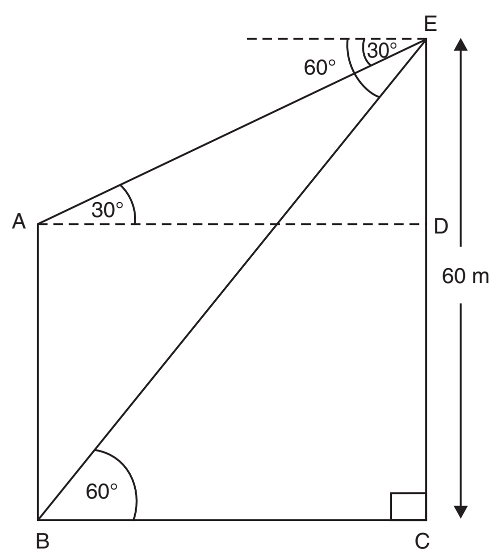
Q. 4. *From the top of a building 60 m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 30° and 60° respectively. Find:*

(i) The horizontal distance between the building and the lamp post.

(ii) The height of the lamp post. [*Take* $\sqrt{3}$ = *1.732*] (CBSE 2012)

Sol. In the figure, let CE be the building and AB be the lamp post

$\therefore$ $CE = 60$ m



In right ΔBCE, we have:

$$\frac{CE}{BC} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \frac{60}{BC} = \sqrt{3}$$

$$\Rightarrow BC = \frac{60}{\sqrt{3}} = \frac{60 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \text{ m}$$

$$\Rightarrow BC = \frac{60\sqrt{3}}{3} = 20\sqrt{3} \text{ m}$$

In right ΔADE, we have:

$$\frac{DE}{AD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{DE}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

[$\because$ $BC = AD = 20\sqrt{3}$ *m*]

$$\Rightarrow DE = \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m}$$

$\therefore$ Height of the lamp post $= AB = CD$

$= CE - DE$

$= 60$ m $- 20$ m

$=$ **40 m.**

Also, the distances between the lamp post and the building

$= 20\sqrt{3}$ m

$= 20 \times 1.732$ m [$\because \sqrt{3} = 1.732$]

$=$ **34.64 m**

Q. 5. *The angle of elevation of a cloud from a point h meters above the surface of a lake is* θ *and the angle of depression of its reflection in the lake is* ϕ*. Prove that the height of the clouds above the lake is* $h\left[\dfrac{\tan\phi+\tan\theta}{\tan\phi-\tan\theta}\right]$. [NCERT Exemplar]

Sol. Let P be the cloud and Q be its reflection in the lake. As shown in the figure, let A be the point of observation such that $AB = h$

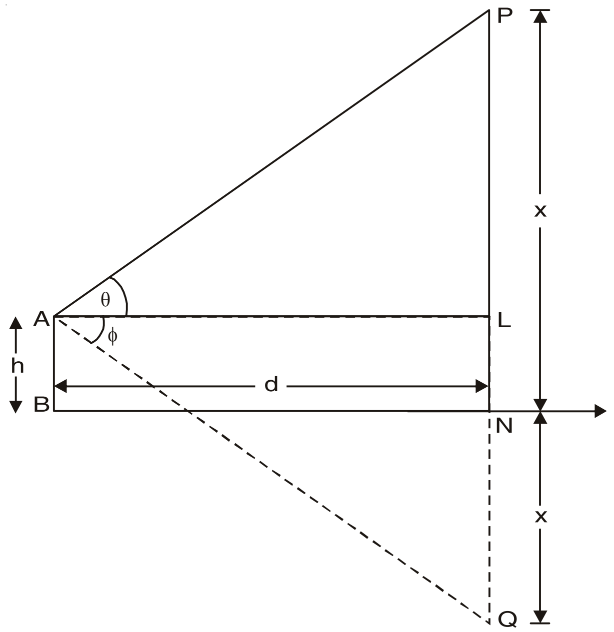
Let the height of the cloud above the lake = x

Let AL = d

From rt ΔPLA, $\tan\theta = \dfrac{PL}{AL} = \dfrac{PN-LN}{AL}$

$$\Rightarrow \quad \tan\theta = \frac{x-h}{d} \quad \text{...(1)}$$

similarly, $$\tan\phi = \frac{x+h}{d} \quad \text{...(2)}$$



From (1) and (2), $\dfrac{\tan\phi}{\tan\theta} = \dfrac{x+h}{x-h}$

or $$\frac{2x}{2h} = \frac{\tan\phi+\tan\theta}{\tan\phi-\tan\theta} \Rightarrow x = h\left[\frac{\tan\phi+\tan\theta}{\tan\phi-\tan\theta}\right]$$

Q. 6. *From a point 100 m above a lake, the angle of elevation of a stationary helicopter is 30° and the angle of depression of reflection of the helicopter in the lake is 60°. Find the height of the helicopter.* (AI CBSE 2008 C)

Sol. In the figure, A is the stationary helicopter and F is its reflection in the lake.
In right $\Delta\ AED$, we have:

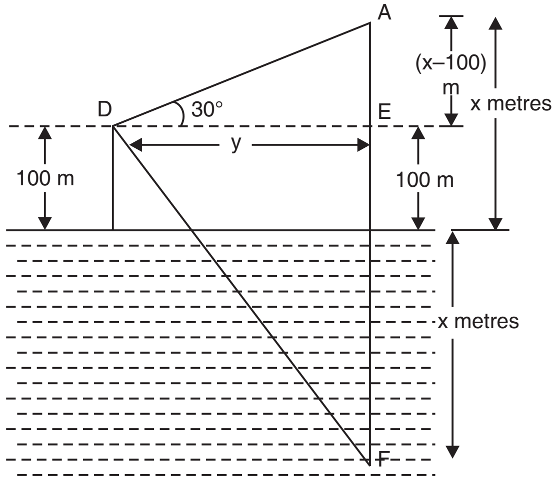
$$\tan 30° = \frac{AE}{DE}$$

But $\tan 30° = \frac{1}{\sqrt{3}}$

$$\therefore \quad \frac{AE}{DE} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{x-100}{y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad y = \frac{x-100}{\sqrt{3}} \quad ...(1)$$



In right ΔDEF, $\tan 60° = \frac{EF}{DE}$

$$\Rightarrow \quad \frac{EF}{DE} = \sqrt{3}$$

$$\Rightarrow \quad \frac{x+100}{y} = \sqrt{3}$$

$$\Rightarrow \quad \sqrt{3}\,y = x + 100$$

But $y = \sqrt{3}\,(x - 100)$

$$\therefore \sqrt{3} \times \sqrt{3}\,(x - 100) = x + 100$$

$$\Rightarrow \quad 3\,(x - 100) = x + 100$$

$$\Rightarrow \quad 3x - 300 - x = 100$$

$$\Rightarrow \quad 2x = 100 + 300$$

$$\Rightarrow \quad 2x = 400$$

$$\Rightarrow \quad x = \frac{400}{2} = 200$$

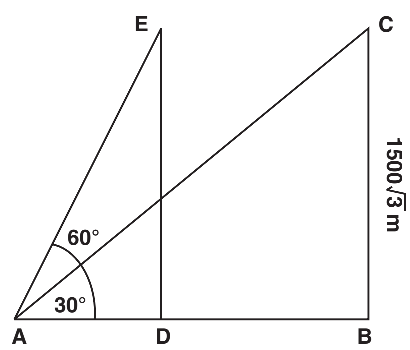
Thus, the height of the stationary helicopter = **200 m.**

Q. 7. *The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of* $1500\sqrt{3}$ *m, find the speed of the aeroplane.* (AI CBSE 2008 C)

Sol. In the figure, let E and C be the two locations of the aeroplane.

Height $BC = ED$

$$= 1500\sqrt{3} \text{ m}$$



In right ΔABC, we have:

$$\frac{BC}{AB} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{1500\sqrt{3}}{AB} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad AB = \sqrt{3} \times 1500 \times \sqrt{3} \text{ m}$$

$$= 3 \times 1500 \text{ m} = 4500 \text{ m}$$

In right ΔADE, we have:

$$\frac{ED}{AD} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad \frac{1500\sqrt{3}}{AD} = \sqrt{3} \qquad [\because ED = BC]$$

$$\Rightarrow \quad AD = \frac{1500\sqrt{3}}{\sqrt{3}} = 1500 \text{ m}$$

Since the distance travelled in 15 seconds $= AB - AD$

$$= 4500 - 1500 = 3000 \text{ m}$$

Since, $\quad \text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

$\therefore$ Speed of the aeroplane $= \dfrac{3000}{15}$ m/s $=$ **200 m/s.**

Q. 8. *A spherical balloon of radius r subtends an angle θ at the eye of the observer. If the angle of elevation of its centre is φ, find the heights of centre of the balloon.* [NCERT Exemplar]

Sol. In the figure, let O be the centre of the balloon, and A be the eye of the observer. r be the radius.

$\therefore \quad OP = r$ and $PAQ = \theta$

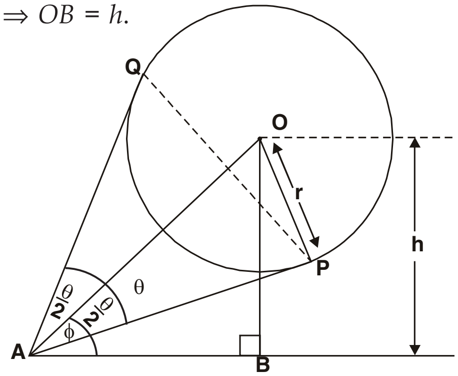
Also, $\quad \angle OAB = \phi$

Let the height of the centre of the balloon be 'h' $\Rightarrow OB = h$.

In $\quad \Delta OAP, \angle OPA = 90°$

$$\Rightarrow \quad \sin\frac{\theta}{2} = \frac{r}{s}, \text{ where } OA = s \quad ...(1)$$

From, ΔOAB, $\sin\phi = \dfrac{h}{s} \quad ...(2)$



Now, from (1) and (2),

$$\frac{\sin\phi}{\sin\frac{\theta}{2}} = \frac{\frac{h}{s}}{\frac{r}{s}} = \frac{h}{s} \times \frac{s}{r} = \frac{h}{r}$$

$$\Rightarrow \quad h = r\left[\frac{\sin\phi}{\sin\frac{\theta}{2}}\right]$$

$$\Rightarrow \quad h = r \,.\, \sin\phi \,.\, \text{cosec}\,\frac{\theta}{2} \qquad \left|\because \frac{1}{\sin\frac{\theta}{2}} = \text{cosec}\,\frac{\theta}{2}\right.$$

Q. 9. *As observered from the top of a light house, 100 m high above sea level, the angle of depression of a ship sailing directly towards it, changes from 30° to 60°. Determine the distances travelled by the ship during the period of observation.* [*Use* $\sqrt{3} = 1.732$] (AI CBSE 2004)

Sol. Let A represents the position of the observer such that

$$AB = 100 \text{ m}$$

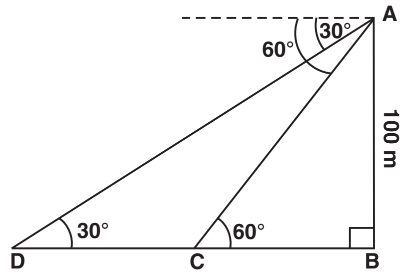
$\therefore$ In right $\Delta\, ABC$, we have

$$\frac{AB}{BC} = \tan 60°$$

$$\Rightarrow \quad \frac{100}{BC} = \sqrt{3} \quad \Rightarrow \quad \sqrt{3}\, BC = 100$$

$$\Rightarrow \quad BC = \frac{100}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} \times \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$$

$$= \frac{100 \times 1.732}{3} = 57.73 \text{ m}$$



In right $\Delta\, ABD$, we have:

$$\frac{AB}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{100}{BD} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BD = \sqrt{3} \cdot 100 = 1.732 \times 100$$

$$\Rightarrow \quad BD = 173.2 \text{ m}$$

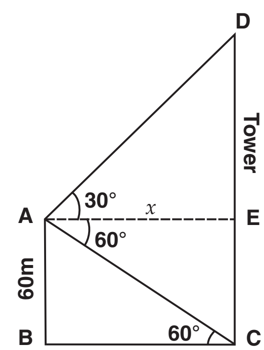
$\therefore$ The distance travelled

$$CD = BD - BC$$

$$= (173.2 - 57.73) \text{ m} = \mathbf{115.47 \text{ m}}$$

Q. 10. *The angles of elevation and depression of the top and the bottom of a tower from the top of a building, 60m high, are 30° and 60° respectively. Find the difference between the heights of the building and the tower and the distance between them.* (CBSE Delhi 2014)

Sol. Let AB is building = 60 m and DC is the tower



In $rt.\ \Delta AED$, $\dfrac{DE}{x} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\therefore \quad x = \sqrt{3} \times DE$ ...(1)

In $rt.\ \Delta ABC$, $\dfrac{AB}{BC} = \tan 60° = \sqrt{3}$

$\Rightarrow \quad \dfrac{60}{x} = \sqrt{3} \Rightarrow x = \dfrac{60}{\sqrt{3}}$ ...(2)

Substituting the value of x from (2) in (1), we have :

$$\sqrt{3}\, DE = \frac{60}{\sqrt{3}} \Rightarrow DE = \frac{60}{\sqrt{3} \times \sqrt{3}} = 20$$

$\Rightarrow$ Difference between the heights of building and tower = 20 m

Distance between the tower and building

$$= x = \sqrt{3} \times 20 = 1.732 \times 20\text{m} = 34.64\text{m}$$

TEST YOUR SKILLS

1. A person standing on the bank of a river observes that the angle of elevation of the top of a tower standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tower and the width of the river. [Use $\sqrt{3}$ = 1.732] [CBSE 2008]

2. A straight highway leads to the foot of a tower. A man standing at top of the tower observes a car at angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

[AI CBSE 2008]

3. An aeroplane, when 3000 m high, passes vertically above another aeroplane at an instant, when the angle of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes.

[Use $\sqrt{3}$ = 1.732] [CBSE 2011, 2012 CBSE 2008 F]

4. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed, in km/hr, of the plane. [CBSE 2008 F]

5. The angle of elevation of a jet fighter from a point A on the ground is 60°. After a flight of 10 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 648 km/hr, find the constant height at which the jet is flying [Use $\sqrt{3}$ = 1.732]

[CBSE 2012] [AI CBSE 2008]

6. The angle of elevation of a jet fighter from a point A on the ground is 60°. After a flight of 10 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 432 km/hr, find the constant height at which the jet is flying [Use $\sqrt{3}$ = 1.732]

[CBSE 2012] [AI CBSE 2008]

7. The angle of elevation of a jet fighter from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 720 km/hr, find the constant height at which the jet is flying. [use $\sqrt{3}$ = 1.732]

[AI CBSE 2008, 2014] [CBSE 2012]

8. A statue 1.46 m tall standing on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal. [Use $\sqrt{3}$ = 1.73]

[CBSE 2008, 2012]

9. From the top of a house, h metres high from the ground, the angles of elevation and depression of the top and bottom of a tower on the other side of the street are θ and ϕ respectively. Prove that the height of the tower is $h(1 + \tan\theta \cot\phi)$. [AI CBSE, 2006, 2007]

10. A window in a building is at a height of 10 m from the ground. The angle of depression of a point P on the ground from the window is 30°. The angle of elevation of the top of the building from the point P is 60°. Find the height of the building. [AI CBSE 2007]

11. A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point A on the ground is 60° and the angle of depression of point A from the top of the tower is 45°. Find the height of the tower. [Take $\sqrt{3}$ = 1.732]

[AI CBSE 2004, 2007]

12. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A gril standing on the roof of 20 m high building finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on opposite sides of the bird. Find the distance of bird from the girl. [CBSE 2007]

13. The angle of elevation of the top of a hill at the foot of the tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill. [AI CBSE 2006 C]

14. The angle of elevation of the top of a tower from a point on the same level as the foot of the tower is 30°. On advancing 150 metres towards the foot of the tower, the angle of elevation becomes 60°. Show that the height of the tower is 129.9 metres. [Use $\sqrt{3}$ = 1.732] [CBSE 2006 C]

15. From a window 15 m high above the ground in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30° and 45° respectively. Show that the height of the opposite house is 23.66 metres. [Take $\sqrt{3}$ = 1.732] [CBSE 2006 C]

16. A man standing on the deck of a ship, which 10 m above the water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill. [CBSE 2012] [AI CBSE 2006]

17. From a point '*A*' on a straight road the angle of elevation of the top of a vertical tower situated on the roof of a vertical building on the sam road is θ. The angle of elevation of the bottom of the tower from a point *B* on the road is again θ. The height of the building is 50 m. If *AB* : *BY* is 2 : 5, where *Y* is the base of building, then show that the height of the tower is 20 m.

18. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the other tower. [NCERT Exemplar]

19. An observer 1.5 m tall is 20.5 m away from a tower 22 m high. Determine the angle of elevation of the top of the tower from the eye of the observer. [NCERT Exemplar]

20. From a point on the ground the angles of elevation of the bottom and top of a transmission tower fixed at the top of 20 m high building are 45° and 60°. Find the height of the tower.

21. The angle of elevation of a cloud from a point 200 m, above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud. [CBSE 2011, 2012]

22. A tree 12 m high is broken by the wind in such a way that its top touches the ground and makes an angle of 60° with the ground. At what height from the bottom the tree is broken by the wind? [CBSE 2011]

23. Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the lighthouse are in the same straight line. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45°. If the height of the lighthouse is 200m, find the distance between the two ships. [Use $\sqrt{3}$ = 1.732] [CBSE 2014]

Hint:

In rt Δ AMP, we have:

$$\frac{PM}{AM} = \tan 60° \Rightarrow \frac{200}{x} = \sqrt{3}$$

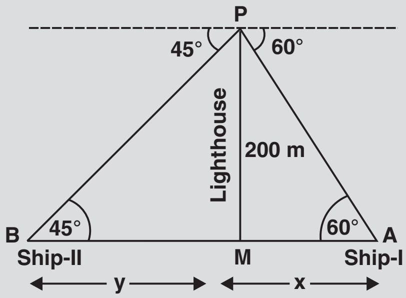
$$\Rightarrow \qquad x = \frac{200}{\sqrt{3}} \qquad ...(1)$$

In rt Δ BMP, we have :

$$\frac{PM}{BM} = \tan 45° \Rightarrow \frac{200}{y} = 1$$

$$\Rightarrow \qquad y = 200 \qquad ...(2)$$

Solving (1) and (2),
we get required distance (x + y) = 315.4 m



24. Two ships are approaching a ligh house from opposite directions. The angles of depression of the two ships from the top of the light house are 30° and 45°. If the distance between the two ships is 100 m, find the height of the light house. [Use $\sqrt{3}$ = 1.732]

[AI. CBSE (Foreign) 2014]

25. The angle of elevation of the top of a tower at a distance of 120m from a point A on the ground is 45°. If the angle of elevation of the top of a flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff. [Use $\sqrt{3}$ = 1.73] [AI. CBSE 2014]

ANSWERS

TEST YOUR SKILLS

1. 34.64 m; 20 m **2.** 3 seconds **3.** 1268 m **4.** 864 km/h **5.** 1558.8 m
6. 1039.2 m **7.** 2598 m **8.** 2 m **10.** 30 m **11.** 6.82 m **12.** $30\sqrt{2}$ m
13. 150 m **15.** 23.6 m **16.** 40 m; $10\sqrt{2}$ m **18.** $10\sqrt{3}$ m , 10 m
19. 45° **20.** $20(\sqrt{3}-1)$ m **21.** 400 m **22.** 5.569 m. **23.** 315.4 m
24. 36.6 m **25.** 87.6 m